KINETIC MODELS OF DISPERSE MEDIA WITH INTERNAL DEGREES OF FREEDOM

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The kinetic approach to the study of gas flows with internal degrees of freedom has been relatively well elaborated [1-3]. It is useful to extend this approach to disperse media, since the number of practically important problems in this area is very extensive [4]. Phenomenological models for the study of disperse media with processes of vibrational relaxation in a gas, on the phase boundary, and inside solid aerosol particles were elaborated in [5-6], and with consideration of phase transitions in [7]. The kinetic model of suspension of matter in gas without internal degrees of freedom was investigated in [8].

Based on the rigorous asymptotic method in [2], equations of motion for suspension of matter in gas were derived in the present study in the case where the gas has one internal degree of freedom, vibrational, for example. The aerosol particles can also be vibrationally excited [5-7].

We will consider the conditions of streamline flow of particles are free-molecule, and the reaction of the gas molecules with the particles is described by a mirror-diffuse Maxwellian scheme. We will neglect the random movement of solid particles. Then the distribution functions of the gas molecules $f_g(t, \mathbf{r}, c_g, E_N)$ in the absence of external forces will satisfy the following kinetic equation

$$\frac{\partial f_g}{\partial t} + \mathbf{c}_g \frac{\partial f_g}{\partial \mathbf{r}} = J_{gg}^{el} + J_{gg}^{in} + I_{gp}, \tag{1}$$

$$J_{gg}^{el} = \sum_{(N)} \int \{f_g f_{g_1}\}\, d\mathbf{P}^{el},\ J_{gg}^{in} = \sum_{(N)} \{f_g f_{g_1}\}\, d\mathbf{P}^{in},\ I_{gp} = (1 - \sigma_i) J_{gp}^{el} + \sigma_i J_{gp}^{in},$$

$$J_{gp}^{el} = n_p r_p^2 \left\{ \sum_{(N)} \int_{(\mathbf{gk}<0)} (\mathbf{gk}) f_g\, d\mathbf{k} - \sum_{(N)} \int_{(\mathbf{g'k}<0)} (\mathbf{g'k}) f'_g \int_{(\mathbf{gk}>0)} \delta\, [\mathbf{g} - (\mathbf{g'} - 2\mathbf{gk})\, \mathbf{k}]\, d\mathbf{g'}\, d\mathbf{k} \right\},$$

$$J_{gp}^{in} = n_p r_p^2 \left\{ \sum_{(N)} \int_{(\mathbf{gk}<0)} (\mathbf{gk}) f_g\, d\mathbf{k} - \sum_{N} \int_{\mathbf{g'k}<0} (\mathbf{g'k}) f'_g \int_{\mathbf{gk}>0} \frac{1}{2\pi} \left(\frac{m}{kT_r}\right)^2 (\mathbf{gk})\, \mathrm{e}^{-\frac{mg^2}{2kT_r}} \mathrm{e}^{-\frac{\mathrm{E}_N}{kT_{ir}}}\, d\mathbf{g'}\, d\mathbf{k} \right\}.$$

Elastic $J_{gg}{}^{el}$ and inelastic $J_{gg}{}^{in}$ integrals of collisions of molecules with each other have the usual Boltzmann form, and the integrals of collisions of molecules with particles of finite radius have the form used in the kinetic theory of gases for studying boundary problems of the interaction with the surface [9]. In Eqs. (1), for simplification of the notation, the terms with rotational degrees of freedom are omitted. As a kinetic boundary condition, it is assumed that part $(1 - \sigma_i)$ of the molecules wihich fall on particles are reflected like a mirror, and the other part σ_i is initially adsorbed and then emerges on the surface with a Maxwell–Boltzmann distribution of translational and vibrational degrees of freedom at different temperatures T_r and T_{ir} [10]. Coefficient σ_i can consist of different parts related to different mechanisms of relaxation on the surface. Each part can in principle be determined by solving discrete quantum mechanical problems of the interaction with the surface, for example, problems of heterogeneous V–T relaxation [11] or resonance V–V exchange by quanta on the surface [6]. The kinetic equation for aerosol particles is

$$\frac{\partial f_p}{\partial t} + \mathbf{c}_p \frac{\partial f_p}{\partial \mathbf{r}} = J_{pp}^{el} + J_{pp}^{in} + I_{pg}. \tag{2}$$

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The reaction of the particles with the gas molecules only results in slow diffusion of the particles in the velocity space due to the large difference in masses, and for this reason, I_{pg} can be represented as [12]

$$I_{pg} = (\partial/\partial \mathbf{c}_p)(\mathbf{K}\partial f_p/\partial \mathbf{c}_p - \mathbf{F}_{pg} f_p)$$

($\mathbf{F}_{pg} = \int mgI_{gp}dc_g$ is the force acting on the particle from the gas molecules, and $\mathbf{K}$ is the tensor of the diffusion coefficients in the velocity space of the individual particle). Integral J_{pp} has the usual Boltzmann form if the random motion of the particles is taken into consideration. There is currently no kinetic description of the evolution of the intramolecular vibrational energy in the particles, and obtaining the structure of J_{pp}^{in} is an independent complex problem. Hence, the conditions of balance of energy fluxes were used in elaborating the phenomenological models in [5, 6] and it was hypothesized that the vibrational energy inside the particles relaxes according to the Landau–Teller law. The basic attention in the present study is focused on investigating the conditions of flow determined by kinetic Eq. (1). We will convert this equation to the dimensionless form by assuming $f_g = \bar{f}_g n/u_0{}^3$, $d\mathbf{k} = d\bar{\mathbf{k}}\pi r_p{}^2$, $\mathbf{g} = \bar{\mathbf{g}}G$, where $\mathbf{g} = \mathbf{c}_g - \mathbf{u}_p$, $G = |\mathbf{u} - \mathbf{u}_p|$, and u, u_p are the average rates of the gas and particles. If the mean free path is l and the characteistic dimension of the problem is L, then after making it dimensionless, we obtain

$$K\left(\frac{\partial f_g}{\partial t} + \mathbf{c}_g \frac{\partial f_g}{\partial r}\right) = J_{gg}^{el} + \alpha J_{gg}^{in} + \frac{K}{Ku}\left[(1-\sigma_i)J_{gp}^{el} + \sigma_i J_{gp}^{in}\right]. \qquad (3)$$

The processes of vibrational relaxation in a gas are assumed to be slow, i.e., $\alpha \sim K$ [1, 2]. In Eq. (3), $K = l/L$ is the Knudsen number, parameter $Ku = r_p/u_0((4/3)\pi r_p{}^3 n_p)\tau$ ($\varepsilon_0 = (4/3)\pi r_p{}^3 n_p$ is the van der Waals number with respect to the particles) characterizes the reaction of the molecules with the aerosol particles. Actually, the value of Ku also has the meaning of the Knudsen number ($u_0\tau \sim L$), but is determined based on the interaction of the gas molecules with the particles. Values of $Ku \gg 1$ are not of great interest, since on the characteristic scale of the problem, the molecules do not collide with the particles in general. For this reason, they practically have the meaning of only $Ku \sim 1$ and $Ku \ll 1$, which are different cases, since the basic integral operator changes in going from $Ku \sim 1$ to $Ku \ll 1$. In the theory of multiatomic gases, the different asymptotics related to separation of small parameters are based on some system of separation of the collision integral. The individual terms of this integral correspond to different types of reaction of the molecules with each other. However, it is important that part of the collision integral corresponding to elastic collisions contains more binary invariants than the entire integral. As a result, the kinetic equation results in a more general solution [1, 2]. Division into individual groups (for example, J_{gg}^{el}, J_{gg}^{in}, and J_{gp}^{el}, J_{gp}^{in}) according to some interactions is also possible in the class of problems examined. It is important that the initial kinetic equation contain the sum of integrals of a different physical nature which do not result in any one formal procedure without loss of the essence of the phenomena examined and which have different systems of binary invariants. The case of $K/Ku \gtrsim 1$ has its own restrictions. The expression $K/Ku \sim 1$ is adequate to $Kn\varepsilon_0 \sim 1$, where $Kn = \ell/r_p$ is the Knudsen number determined with respect to the particle radius. If $\varepsilon_0 \sim 1$, then it is necessary to consider the "constraint" of the medium investigated and it is not possible to neglect the pressure of the particles, since $P_p/P_g \propto \varepsilon_0$ [13]. In our case, $\varepsilon_0 \ll 1$, and for this reason, in agreement with the above hypothesis, the condition of streamline flow of the particle should be free-molecule flow. We will look for the solution of Eq. (3) in the form of the series $f_g = f_g{}^{(0)} + Kf_g{}^{(1)} + \ldots$ ($K \ll 1$). If $Ku \lesssim 1$ and $K/Ku \ll 1$, which occurs in problems of laser gas dynamics [6], then regardless of the value of σ_i, $J_{gg}^{el} = 0$ will be the basic integral operator. In the kinetic theory of gases, this corresponds to the so-called relaxation case [2], i.e., the comparatively rare collision of molecules with particles on gas dynamics time scales and distances. If these collisions become as frequent as the collisions of the molecules with each other, i.e., $Ku \ll 1$, $K/Ku \sim 1$, the basic integral operator changes, becoming $J_{gg}^{el} + J_{gp}^{el} + J_{gp}^{in} = 0$ (so-called equilibrium case in kinetic theory of gases [2]). We will successively examine these situations.

I. $K \ll 1$, $Ku \sim 1$, $\sigma_i \lesssim 1$. Substituting the series expansion of the distribution function f_g with respect to the number K in Eq. (3), in the zero approximation we obtain $J_{gg}^{el} = 0$. A two temperature Maxwell–Boltzmann distribution function is the solution of this equation [1]

$$f_g^{(0)} = n\left(\frac{m}{2\pi kT_g}\right)^{3/2}\left[\sum_{(N)}\exp\left(-\frac{E_N}{kT_{ig}}\right)\right]^{-1}\exp\left[-\frac{m(\mathbf{c}_g-\mathbf{u})^2}{2kT_g}\right]\exp\left(-\frac{E_N}{kT_{ig}}\right). \tag{4}$$

If kinetic Eq. (3) in the zero approximation is

$$\frac{\partial f_g^{(0)}}{\partial t} + \mathbf{c}_g\frac{\partial f_g^{(0)}}{\partial \mathbf{r}} - J_{gg}^{in} - \left[(1-\sigma_i)J_{gp}^{el}\left(f_g^{(0)}\right) + \sigma_i J_{gp}^{in}\left(f_g^{(0)}\right)\right] = 0$$

we will successively multiply by invariants 1, mc_g, $mc_g^2/2$, E_N of operator J_{gg}^{el}, integrate with respect to the velocity space, and sum with respect to the internal energy space, for which a set of values of the harmonic oscillator energy with an infinite number of levels will subsequently be used, and we obtain a system of Euler hydrodynamic equations

$$\frac{d\rho}{dt} + \rho\nabla u = 0,\ \rho\frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{F}_{gp},\ \frac{d\overline{E}}{dt} + kT_g\frac{\partial \mathbf{u}}{\partial \mathbf{r}} = Q_{gp} + \mathbf{F}_{gp}\mathbf{u}, \tag{5}$$

$$\overline{E} = \frac{3}{2}kT_g + \varepsilon_N,\ \varepsilon_N = \sum_{(N)}\exp(-E_N/kT_{ig})\Big/\sum_{(N)}\exp(-E_N/kT_{ig}),$$

$$n\frac{d\varepsilon_N}{dt} = \frac{\varepsilon_N(T_g)-\varepsilon_N(T_{ig})}{\tau_{ig}} + \frac{\varepsilon_N(T_{ig})-\varepsilon_N(T_{ir})}{\tau_i^*}$$

$$\Bigg(\mathbf{F}_{gp} = \sum_{(N)}\int m\mathbf{c}_g I_{gp}\,d\mathbf{c}_g \text{ — strength of interphase interaction,}$$

$$Q_{gp} = \sum_{(N)}\int\left(\frac{m\mathbf{c}_g^2}{2} + E_N\right)I_{gp}\,d\mathbf{c}_g \text{ — interphase heat exchange}\Bigg).$$

The concrete expressions for $\mathbf{F}_{gp}$ and Q_{gp} can be borrowed from the calculations for the streamline flow of a sphere with free-molecule flow [14, 15]. The pulse accommodation coefficients should be understood as σ_i in the expressions for $\mathbf{F}_{gp}$. The major possibility of V–T exchange on the surface complicates the calculation of the heat fluxes Q_{gp}. The possibility of V–T transitions can be grossly taken into consideration by introducing additional accommodation coefficients [16]. The right part of the relaxation equation in system (5) contains terms which account for relaxation processes in the gas and on the surface. The corresponding vibrational relaxation times τ_{ig}, τ_i^* were determined in [2, 17]. The special case of system (5) for flow after a direct shock wave of a vibrationally relaxing gas with particles "frozen" in it was examined in [17].

In a first approximation with respect to K, kinetic Eq. (3) is

$$\frac{\partial f_g^{(0)}}{\partial t} + \mathbf{c}_g\frac{\partial f_g^{(0)}}{\partial \mathbf{r}} - I_{gp}\left(f_g^{(0)}\right) - J_{gg}^{in}\left(f_g^{(0)}\right) = J_{gg}^{el}\left(f_g^{(1)}\right). \tag{6}$$

Substituting function (4) in (6) and excluding the partial derivatives with respect to time with (5), we obtain

$$f_g^{(0)}\Bigg\{\left(\frac{mC_g^2}{2kT_g} - \frac{5}{2}\right)\frac{\partial \ln T_g}{\partial \mathbf{r}}\mathbf{C}_g + \frac{m}{kT_g}\left(\overline{\mathbf{C}_g\overset{\circ}{\mathbf{C}}_g}\colon \frac{\partial \mathbf{u}}{\partial \mathbf{r}}\right) + \frac{(E_N-\varepsilon_N)}{kT_{ig}}\frac{\partial \ln T_{ig}}{\partial \mathbf{r}}\mathbf{C}_g +$$

$$+ \frac{d\varepsilon_N}{dt}\left[\left(\frac{1}{kT_g} - \frac{mC_g^2}{3k^2T_g^2}\right) + \frac{(E_N-\varepsilon_N)}{kT_{ig}^2 c_{vi}(T_{ig})}\right]\Bigg\} - J_{gg}^{in}\left(f_g^{(0)}\right) - I_{gp}\left(f_g^{(0)}\right) = J_{gg}^{el}\left(f_g^{(1)}\right).$$

Integral $I_{gp}(f_g^{(0)})$ can be represented in an expansion with respect to irreducible tensors:

$$I_{gp}\left(f_g^{(0)}\right) = f_g^{(0)}\left[L_0 + \mathbf{C}_g\mathbf{L}_1 + \left(\frac{mC_g^2}{2kT_g} - \frac{3}{2}\right)L_2 + \frac{m}{kT_g}\overline{\mathbf{C}_g\overset{\circ}{\mathbf{C}}_g}\colon \overline{\overset{\circ}{\mathbf{L}}_2} + \mathbf{C}_g\left(\frac{mC_g^2}{2kT_g} - \frac{5}{2}\right)\mathbf{L}_3 + \mathbf{C}_g\frac{(E_N-\varepsilon_N)}{kT_{ig}}\mathbf{L}_4 + \ldots\right], \tag{7}$$

where $\mathbf{L}_1$, L, $\mathbf{L}_3$, and the other coefficients are calculated by multiplying both parts of series (7) by the corresponding factor with subsequent integration with respect to the rate and sumamtion with respect to the internal energy space, but they are somewhat unwieldy. For example,

$$\mathbf{L}_3 = \frac{4}{5n}\left(\frac{m}{2kT_g}\right)\sum_{(N)}\int C_g^2\left(\frac{mC_g^2}{2kT_g} - \frac{5}{2}\right)I_{gp}\left(f_g^{(0)}\right)d\mathbf{c}_g =$$

$$= -\frac{4}{5}\pi n_p r_p^2\mathbf{\Omega}\left\{\frac{\exp(-\Omega^2)}{\sqrt{\pi}}\left(\Omega^2 + \frac{9}{8\Omega^2} + \frac{5}{2}\right) + \left[\left(\Omega^3 + 3\Omega + \frac{15}{8\Omega} - \frac{9}{16\Omega^3}\right) + \frac{5}{2\gamma}\left(\Omega + \frac{1}{5\Omega} + \frac{3}{20\Omega^3}\right)\right]\operatorname{erf}\Omega + \sqrt{\frac{\pi}{\gamma}}\left(\Omega^2 + \frac{5}{6\gamma} + \frac{5}{6}\right)\right\}$$

$$\left(\mathbf{\Omega} = \sqrt{\frac{m}{2kT_g}}(\mathbf{u}-\mathbf{u}_p),\ \gamma = \frac{T_g}{T_p},\ \sigma_i = 1\right).$$

The structural form of the distribution function of the first approximation is the same as in [18, 2]:

$$\varphi = -\mathbf{A}_1(\nabla \ln T_g - \mathbf{L}_3) - \mathbf{A}_2(\nabla \ln T_{ig} - \mathbf{L}_4) - \mathbf{B} : (\nabla \mathbf{u} - \overline{\overset{\circ}{\mathbf{L}}}_2) - G.$$

Integral equations similar to those reported in [2, 19], but with consideration of the terms $I_{gp}(f_g^{(0)})$ responsible for heterogeneous processes, take place relative to unknowns $\mathbf{A}_1$, $\mathbf{A}_2$, ... The dissipative coefficients can be determined from the solution of these equations with the procedure described in detail in [2, 19]. In the case of the disperse medium examined, these coefficients will apparently coincide with those obtained in [19] for a vibrational relaxing gas, since the structures of the basic integral operators coincide. The effect of the particles is only reflected in the structure of the flow terms. The heat flux and pressure tensor are written as

$$\mathbf{q} = \sum_{(N)} \int f_g^{(0)} \left(\frac{mC_g^2}{2} + E_N\right) \left[\left(\mathbf{A}_1 \frac{\partial T_g}{\partial \mathbf{r}} - \mathbf{L}_3\right) + \left(\mathbf{A}_2 \frac{\partial T_{ig}}{\partial \mathbf{r}} - \mathbf{L}_4\right)\right] \mathbf{C}_g d\mathbf{c}_g =$$

$$= -\lambda_t\left(\frac{\partial T_g}{\partial \mathbf{r}} - \mathbf{L}_3\right) - \lambda_i\left(\frac{\partial T_{ig}}{\partial \mathbf{r}} - \mathbf{L}_4\right),$$

$$\mathbf{P} = \sum_{(N)} \int m f_g^{(0)} \mathbf{C}_g \mathbf{C}_g \overline{\overset{\circ}{\mathbf{B}}} : (\nabla \mathbf{u} - \overline{\overset{\circ}{\mathbf{L}}}_2)\, d\mathbf{c}_g = -\mu_g(\nabla \mathbf{u} - \overline{\overset{\circ}{\mathbf{L}}}_2)$$

(λ_t, λ_i are thermal conductivity coefficients for translational and vibrational degrees of freedom [19], μ_g is the shear viscosity coefficient).

II. $K \ll 1$, $\alpha \sim K$, $Ku \ll 1$, $K/Ku \sim 1$. In this case, in the zero approximation

$$J_{gg}^{el}(f_g^{(0)}) + I_{gp}(f_g^{(0)}) = 0, \tag{8}$$

i.e., the effect of the particles is exercised on the scale of the mean free path of the molecules ℓ. This effect can be assessed, for example, by examining the problem of a shock wave in a disperse medium, for simplicity assuming that there is no relative motion between gas and particles. Then for the change in the temperature in the relaxation zone [17], we obtain

$$u\frac{dT}{dx} = \frac{T - T_w}{(\sigma_i Z^{-1})(1 + n\bar{v}\tau_a/\sigma_0)} \tag{9}$$

($Z^{-1} = (\pi r_p^2 n_p \bar{v})^{-1} = (\varepsilon_0 \bar{v}/r_p)^{-1}$, τ_a is the adsorption time, and $\sigma_0 = 10^{15}$ cm^{-2}). The degree of filling of the surface by adsorbed molecules is usually low and factor $n\bar{v}\tau_a/\sigma_0$ can be neglected. If the temperature behind the wave front is $T = T_g$, the solution of Eq. (9) will be $T(x) = T_w + (T_g - T_w)\exp(-x/(\sigma_i L Ku))$. It follows that for $x \sim l$ and $\sigma_i \approx 1$, when $T(x) = T_w + (T_g - T_w)\exp(-K/Ku)$, in the case of $K/Ku \gtrsim 1$, the temperature of the gas T can differ from the value of T_g, only determined by the gaseous medium [4], by a finite value. The principal term of basic integral operator (8) is I_{gp}, since it determines the equilibrium state between gas and particles. The so-called principle of duality [9], which is the analog of the principle of detailed balance in the kinetic theory of gases, but for collisions of molecules with the surface, also holds for integral I_{gp}. This principle also holds in the presence of internal degrees of freedom [20]. Using the H theorem [18], it is possible to show that the H function of each term in (8) is less than or equal to zero. For arbitrary n_p and σ_i, the condition is satisfied if I_{gp} and J_{gg}^{el} become zero. For diffuse reflection, $I_{gp} = 0$ in the case where the distribution of the dropping molecules is locally Maxwellian with a temperature equal to the surface temperature $T_w = T_p$ [9], and for mirror reflection, in the absence of relative movement of the gas and particles. For this reason, for the zero-approximation distribution function, we will write

$$f_{gp}^{(0)} = n\left(\frac{m}{2\pi k T_p}\right)^{3/2} \left[\sum_{(N)} \exp\left(-\frac{E_N}{kT_{ip}}\right)\right]^{-1} \exp\left[-\frac{m(\mathbf{c} - \mathbf{u}_p)^2}{2kT_p}\right] \exp\left(-\frac{E_N}{kT_{ip}}\right).$$

This function also satisfies the conditions $J_{gg}^{el} = 0$.

Macroscopic parameters T_p, T_{ip}, ... in the general case are determined with consideration of the "hydrodynamics" of the particles, i.e., a system of equations which follows from kinetic Eq. (2). If the processes of vibrational relaxation inside and on the surface of isolated particles are frozen, complete equilibrium on hydrodynamic scales (~L) is established due to processes of V–T relaxation in the gas and on the surface of the particles.

The kinetic coefficients determined by the standard procedure in [2, 18, 19], in contrast to case I, will be a function of the collisions of the molecules with the particles. In particular, for viscosity μ_{gp} and thermal conductivity λ_{tp} coefficients, when $\sigma_i = 1$, we obtain

$$\mu_{gp} = \mu_g \left(1 + \frac{2n_p r_p^2}{3n\Omega_1^2(2)}\right)^{-1} = \mu_g \left(1 + a\frac{K}{Ku}\right)^{-1}, \tag{10}$$

$$\lambda_{tp} = \lambda_t \left(1 + \frac{n_p r_p^2}{4n\Omega_1^2(2)}\right)^{-1} = \lambda_t \left(1 + a\frac{K}{Ku}\right)^{-1}$$

($\Omega_1{}^2(2)$ is a bracket integral identical to the integrals determined in [18], and $a \sim 1$). The internal degrees of freedom do not affect coefficients μ_{gp} and λ_{tp}, and for this reason they should coincide with the corresponding coefficients for suspension of matter in gas consisting of a monoatomic gas and unexcited particles. Similar suspension of matter in gas was studied previously in [8], but with the generalized Chapman–Enskog method [21]. The following expression was obtained for the viscosity coefficient:

$$\mu'_{gp} = \mu_g \left\{1 + \frac{n_p r_p^2}{n}\sqrt{\frac{2kT_g}{m}}\,\frac{8\exp(-\Omega^2) + \left(8\Omega^2 + 8 - \frac{2}{\gamma}\right)\frac{\sqrt{\pi}}{\Omega}\operatorname{erf}\Omega}{12\Omega_1^2(2)}\right\}^{-1}. \tag{11}$$

The results of the calculations of the viscosity coefficients with (11) are shown in Fig. 1. Note that beginning with some values of temperature factor γ, the viscosity coefficient becomes negative in a certain region of a change in the value of Ω. This is observed even for small values of the ratio K/Ku, where the additions to the transfer coefficients caused by the presence of another phase should be small (~O(K/Ku)).

We will attempt to explain what caused this situation. According to the generalized method, construction of a composite solution of the kinetic equations suitable for any values of ratio K/Ku is possible.[*] Since the concentrations of particles n_p and the other values contained in Ku ($Ku = l_g/l_{gp}$, $l_{gp} = 1/(\pi r_p{}^2 n_p)$) can change independently, ratio K/Ku can actually be arbitrary. Then applying the algorithm of the generalized method to the problem examined, we find [8] that the local equilibrium solution should be determined from the condition $J_{gg} = 0$ regardless of the value of K/Ku,[†] as in a pure monoatomic gas. As a consequence, in the principal approximation, the particles do not affect the parameters of the gas phase even for K/Ku > 1, although exchange of the kinetic energy of the molecules with the surface of the particles takes place virtually after one collision.

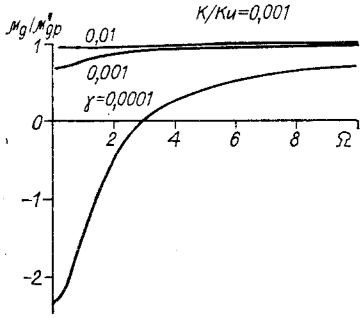

Fig. 1

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[*] In gases with internal degrees of freedom, probability α of inelastic energy exchange [21] in molecular collisions plays the role of parameter K/Ku. The corresponding collision integrals are of the Boltzmann type. In the given case, the structure of J_{gp} is slightly different, but this is not an obstacle to the possible use of the generalized method, as demonstrated in [8].

[†] In the rigorous methods in [1-3, 22], the solution determined by the condition $J_{gg} = 0$ holds for small values of α or a ratio K/Ku, but it is also practically applicable for $\alpha \lesssim 1$ in most cases.

Because of the physical impossibility of this situation, a thin transition layer of the order of magnitude of the mean free path in which $T_g \to T_p$ should arise, i.e., a region corresponding to kinetic conditions not described by hydrodynamic equations. In the rigorous methods based on distinct separation of the asymptotic scales, such conditions have either been studied separately [22] or are considered covered ground [2]. Separation into "fast" $\sim l$ and "slow" $\sim L$ processes (or into equilibrium and relaxation processes) can be classified with the Knudsen number $K = l/L \ll 1$. A continuous change in any of them in the entire possible range means going from the scale of L to l or vice versa, i.e., it includes the condition where $K = 1$. However, in the hydrodynamic statement, this transition is not possible without interruption. This also predetermined the practice of examining different asymptotic cases, including the so-called equilibrium and relaxation cases, in the study of gases with internal degrees of freedom [1-3, 16]. In the generalized method, the thin transitional layers are not explicit, but if necessary are included in the algorithm of the method in accordance with the basic goal — to obtain a continuous solution in the entire region of the change in the parameter. Physically, the discontinuity problem is formally transformed into a continuous problem, which also results in hydrodynamic equations with macroparameters which do not correspond to the real physical situation and generally speaking, in other transfer coefficients.

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CONSIDERATION OF VARIABLE VISCOSITY IN THE DYNAMICS OF SOILS AND POROUS MULTICOMPONENT MEDIA

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Analysis of tests indicates that the bulk viscosity of soils, rocks, ice, and snow is not a constant of the medium, but varies in the loading process.

The model of a solid nonlinear viscoplastic multicomponent medium, intended to describe wave processes [1], is refined below by the introduction of variable bulk viscosity. It is assumed that the viscosity varies (increases) as the state of the medium shifts from a dynamic to a static bulk-compression diagram under load. With this approach, wave processes are described by a system of hyperbolic quasi-linear equations in partial derivatives, just as for constant viscosity. This makes it possible to solve a broad class of wave problems.

1. Determination of Bulk Viscosity from Experimental Data. The variation in the bulk-viscosity coefficient η_L as a function of the loading regime has been noted in many experimental studies. Lyakhov [2] indicates that the η_L of a sandy soil increases by a factor of five as the rise time of the blast loading increases by a factor of three. In rocks [3], the viscosity increases by a factor of 10 as the duration of the load increases under the same stress level. At the same time, the viscosity decreases with increasing stress. For similar maximum stresses in loess and clayey soils, η_L increases by several factors as the loading rate decreases [1, 4].

Let us examine the results of tests [5, 6] in which the spread velocity c and absorption decrement Δ of plane waves of different frequency f, created by a sinusoidal load in frozen soils and in ice, and from which it is possible to determine the viscosity, and the law governing its variation as a function of loading regime, were determined.

The tests corresponded to small strains ε ranging from 10^{-7} to $5\cdot10^{-4}$. In this region, the nonlinearity of the limiting compression diagrams and strain irreversibility can be neglected, and the model of a standard linear body can be used, if the viscosity is considered constant. In the case of a uniaxial strain state, the equation of compression and unloading assumes the form

$$\dot{\varepsilon} - \frac{\dot{\sigma}}{E_D} - \frac{\mu(\sigma - E_s\varepsilon)}{E_S} = 0, \tag{1.1}$$

where $E_D = c_D{}^2\rho_0$, and E_S are the limiting dynamic (when $\dot{\sigma} \to \infty$) and static (when $\dot{\sigma} \to 0$) compression moduli, respectively, μ is a viscosity parameter, c_D is the wave velocity when $f \to \infty$, ρ_0 is the initial density of the medium, and σ is the stress component in the direction of wave propagation. The factor η_L is linked to the viscosity parameter in the following manner:

$$\eta_L = E_S(E_D - E_S)/E_D\mu = E_D(\gamma - 1)/\gamma^2\mu,\ \ \gamma = E_D/E_S. \tag{1.2}$$

During wave propagation, nonsteady oscillations, which convert gradually to steady-state oscillations, develop in the medium. The extinction rate of the amplitude σ of the steady-

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